Relationship between Giffen goods and complementary goods

Consider an economy where there are only 2 goods, x_1 and x_2 with corresponding prices and income p_1, p_2 and m. If one of these goods is a Giffen good, demonstrate that the other good must necessarily be complementary if the conditions of weak monotonicity are met.

Solution

Since weak monotonicity is satisfied, we can assume that the budget constraint will be met with equality at the optimum.

$$p_1 x_1^* + p_2 x_2^* = m$$

Assuming that good x_1 is a Giffen good, now differentiating totally:

$$\frac{\partial x_1^*}{\partial p_1} p_1 + x_1^* + p_2 \frac{\partial x_2^*}{\partial p_1} = 0$$

We know that the first two terms must be positive as good x_1 is a Giffen good, therefore, to maintain equality, it must be that $\frac{\partial x_2^*}{\partial p_1} < 0$, which occurs if good 2 is complementary to good 1.